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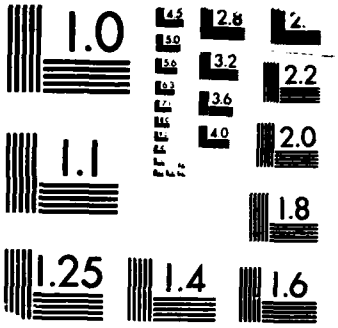

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Xiru Chen and Yuehua Wu

Center for Multivariate Analysis
University of Pittsburgh

Technical Report No. 87-47

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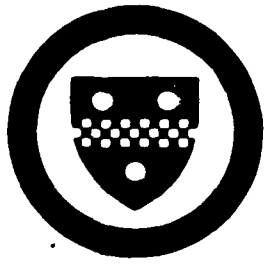

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STRONG LAW FOR MIXING SEQUENCE *

Xiru Chen and Yuehua Wu

ABSTRACT

In this note we present some theorems on the strong law for the mixing sequence which is not necessarily stationary, and the mixing coefficient involving only a pair of variables in the sequence.

AMS 1980 Subject Classifications: Primary 60F15.

Key words and phrases: mixing coefficient, stationary sequence, strong law of large numbers.

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1. INTRODUCTION

In this article we present some results concerning the strong law of a mixing sequence $\{X_n, n \geq 1\}$. We do not assume that $\{X_n\}$ is stationary, and we use mixing coefficients involving only a pair of variables X, Y (in that order): The Rosenblatt mixing coefficient

$$\alpha(X,Y) = \sup\{|P(X \in A, Y \in B) - P(X \in A)P(Y \in B)|: A \in \mathcal{B}', B \in \mathcal{B}'\}$$

and the Ibragimov mixing coefficient

$$\beta(X,Y) = \sup\{|P(Y \in B|X \in A) - P(Y \in B)|: A \in \mathcal{B}', B \in \mathcal{B}', P(X \in A) > 0\}$$

where $\mathcal{B}'$ is the σ-field of all Borel sets in R'.

THEOREM 1. Suppose that $\{X_n, n \geq 1\}$ is a sequence of random variables, and for some $p > 1$ the following conditions are satisfied:

1°. $$\sup_n E|X_n|^p < \infty. \tag{1}$$

2°. There exists $\varepsilon > 0$ such that as $|i - j| \to \infty$,

$$\alpha(X_i,X_j) \leq \rho(|i - j|) = \begin{cases} O(|i - j|^{-p/(2p-2)-\varepsilon}), & 1 < p < 2, \\ O(|i - j|^{-2/p-\varepsilon}), & p \geq 2. \end{cases} \tag{2}$$

Then

$$\lim_{n\to\infty}(S_n - ES_n)/n = 0, \quad \text{a.s.} \tag{3}$$

Here and in the sequel $S_n = \sum_{i=1}^n X_i$.

THEOREM 2. Suppose that $\{X_n, n \geq 1\}$ is a sequence of random variables, or and one of the following conditions are satisfied:

(I) $$\sum_{n=1}^{\infty} \text{var}(X_n)/n^2 < \infty, \qquad \sup_n E|X_n| < \infty,$$

and

$$\beta(X_i,X_j) \leq \mu(|i - j|), \qquad \sum_{n=0}^{\infty} \mu^{1/2}(n) < \infty; \tag{4}$$

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(II) $\sup_n \text{var}(X_n) < \infty$ and there exists $\varepsilon > 0$ such that

$$\sum_{i=1}^{n} \mu^{1/2}(i) = O\left(n/(\log n)^{1+\varepsilon}\right); \tag{5}$$

(III) (4) holds, $X_1, X_2, \ldots$ are identically distributed and $E|X_1| < \infty$ (the existence of variance is not assumed). Then (3) is true.

Remarks:

1. Part (I) of Theorem 2 can be compared with a result of Blum *et al* [1], who assumes that $\{X_n\}$ is a *-mixing sequence instead of (4). Note that this assumption does not follow from (4). We can easily construct a pairwise independent sequence which is not *-mixing.

2. Parts (I) and (II) of Theorem 2 can also be compared with some results (see Theorem 3.7.2 and Theorem 3.7.4 of Stout [5]) derived from Serfling [4]. The conditions of these results involve correlation coefficients between two variables in the sequence.

3. Part (III) of Theorem 2 extends Theorem 1 of Etemadi [2]. The assumption that $\{X_i\}$ is identically distributed can be somewhat relaxed, for example, it can be replaced by the condition that there exists a random variable Y such that $P(|X_n| \geq x) \leq P(|Y| \geq x)$ for all $n \geq 1$ and $x \geq 0$. We also mention a related result of Blum *et al* [1] Theorem 1. They assume that $\{X_n\}$ is identically distributed, the distribution of X_1 has a moment generating function in the neighborhood of zero and that $\{X_n\}$ is *-mixing. Under these more stronger conditions they prove that $P(|S_n - ES_n|/n \geq \varepsilon)$ tends to zero exponentially.

2. PROOF OF THE THEOREMS

In deducing our results we shall borrow a trick from Etemadi [2]. The following well-known facts concerning $\alpha(X,Y)$ and $\beta(X,Y)$ will be used:

$$|\mathrm{cov}(X,Y)| \le 10\big(\alpha(X,Y)\big)^{\delta/(2+\delta)}(E|X|^{2+\delta}E|Y|^{2+\delta})^{1/(2+\delta)}, \quad \delta > 0 \tag{6}$$

$$|\mathrm{cov}(X,Y)| \le 2\big(\beta(X,Y)\mathrm{var}(X)\mathrm{var}(Y)\big)^{1/2}. \tag{7}$$

For a proof, see Ibragimov and Linnik [3]. Also it is trivially true that

$$\alpha\big(XI_C(X),\ YI_D(Y)\big) \le \alpha(X,Y), \qquad \beta\big(XI_C(X),\ YI_D(Y)\big) \le \beta(X,Y) \tag{8}$$

$$\alpha(X-a,\ Y-b) = \alpha(X,Y), \qquad \beta(X-a,\ Y-b) = \beta(X,Y), \tag{9}$$

where C and D are Borel sets in R^1 and a, b are constants.

Proof of Theorem 1. In view of (9), by defining $X_n^+ = X_n I(X_n > 0)$, $X_n^- = -X_n I(X_n \le 0)$, $n \ge 1$, we can assume without loss of generality that $X_n \ge 0$, $n \ge 1$. Define

$$\begin{aligned} Y_n &= (X_n - EX_n)I(|X_n - EX_n| < n^{1/p+\varepsilon_1}), \quad n \ge 1, \\ S_n^* &= \sum_{i=1}^{n}(Y_i - EY_i), \end{aligned} \tag{10}$$

where $\varepsilon_1 > 0$ is a constant to be chosen later.

From condition (1) we have $\sum_{n=1}^{\infty} P(X_n - EX_n \ne Y_n) < \infty$ and $\lim_{n\to\infty} EY_n = 0$. Therefore, (3) is equivalent to

$$\lim_{n\to\infty} S_n^*/n = 0, \quad \text{a.s.} \tag{11}$$

Now fix $\alpha > 1$ and let $k_n = [\alpha^n]$. For positive integer m sufficiently large, there exists n such that $k_n \le m < k_{n+1}$, and $n \to \infty$ as $m \to \infty$. From (1) we have

$$\sup_n E|Y_n| \equiv C < \infty. \tag{12}$$

Here and in the sequel C is an unimportant constant which is allowed to change. Since $Y_n \geq 0$, it follows that

$$S^*_m - S^*_{k_n} \geq -(m - k_n)C, \quad \text{when} \quad S^*_m < S^*_{k_n},$$

$$S^*_m - S^*_{k_n} \leq S^*_{k_{n+1}} - S^*_{k_n} + (k_{n+1} - m)C, \quad \text{when} \quad S^*_m \geq S^*_{k_n}.$$

Hence

$$|S^*_m/m - S^*_{k_n}/k_n| \leq \left|\frac{k_{n+1}}{k_n}\frac{S^*_{k_{n+1}}}{k_{n+1}} - \frac{S^*_{k_n}}{k_n}\right| + \frac{k_{n+1} - k_n}{k_n} C. \tag{13}$$

From (13) it follows that if we have shown that

$$\lim_{n\to\infty} S^*_{k_n}/k_n = 0, \quad \text{a.s.} \tag{14}$$

Then we would have

$$\lim_{m\to\infty} \sup |S^*_m/m| \leq (\alpha - 1)C, \quad \text{a.s.}$$

For any $\alpha > 1$, hence (11).

By Borel-Cantelli lemma, in order to prove (14), we have only to show that

$$\sum_{n=1}^{\infty} \text{var}(S^*_{k_n})/k_n^2 < \infty. \tag{15}$$

By (6), (8) and (9), we have for any $\delta > 0$:

$$\text{Var}(S^*_{k_n}) = \sum_{k,j=1}^{k_n} \text{cov}(Y_i, Y_j)$$

$$\leq C \sum_{i,j=1}^{k_n} \left(\alpha(X_i, X_j)\right)^{\delta/(2+\delta)} (E|Y_i|^{2+\delta} E\{Y_j|^{2+\delta})^{1/(2+\delta)}. \tag{16}$$

From (1) it follows that

$$E|Y_n|^{2+\delta} \le Cn^{(2+\delta-p)(1/p+\varepsilon_1)}, \quad n = 1,2,\ldots . \tag{17}$$

First consider the case $p > 2$. From (2), (16) and (17) we obtain

$$\begin{aligned} \operatorname{var}(S^*_{k_n}) &\le C \sum_{i,j=1}^{k_n} \left(\alpha(X_i,X_j)\right)^{\delta/(2+\delta)} (ij)^{(2+\delta-p)(1/p+\varepsilon_1)/(2+\delta)} \\ &\le C \sum_{i,j=1}^{k_n} \left(\alpha(X_i,X_j)\right)^{\delta/(2+\delta)} i^{2(2+\delta-p)(1/p+\varepsilon_1)/(2+\delta)} \\ &\le C \sum_{i,j=1}^{k_n} i^{-(2/p+\varepsilon)\delta/(2+\delta)} \sum_{i=1}^{k_n} i^{2(2+\delta-p)(1/p+\varepsilon_1)/(2+\delta)} . \end{aligned} \tag{18}$$

Noticing $2/p < 1$, we can assume that $2/p + \varepsilon < 1$. Hence from (18) we have

$$\operatorname{var}(S^*_{k_n}) \le Ck_n^{-(2/p+\varepsilon)\delta/(2+\delta) + 2(2+\delta-p)(1/p+\varepsilon_1)/(2+\delta) + 2} . \tag{19}$$

This inequality holds for any $\delta > 0$. Now we choose $\varepsilon_1 \in (0,\varepsilon/2)$, then

$$\lim_{\delta\to\infty}\{-(2/p+\varepsilon)\delta/(2+\delta) + 2(2+\delta-p)(1/p+\varepsilon_1)/(2+\delta)\} = -\varepsilon + 2\varepsilon_1 \equiv \eta < 0.$$

Therefore, choosing δ sufficiently large, from (19) we obtain $\operatorname{var}(S^*_{k_n}) \le Ck_n^{2-\eta}$. Hence (15) is true in view of $\sum_{n=1}^{\infty} k_n^{-\eta} < \infty$.

Next assume that $p = 2$. Again, choose $\varepsilon_1 \in (0,\varepsilon/2)$. Choose $\delta > 0$ sufficiently small, such that $(1+\varepsilon)\delta/(2+\delta) < 1$. We still have (19), with $p = 2$. Since

$$-(1+\varepsilon)\delta/(2+\delta) + 2\delta(1/2+\varepsilon_1)/(2+\delta) = -(\varepsilon - 2\varepsilon_1)\delta/(2+\delta) < 0,$$

(15) holds again.

Finally, consider the case $1 < p < 2$. In this case we have, instead of (18),

$$\operatorname{var}(S^*_{k_n}) \le C \sum_{i=1}^{k_n} i^{-\left(p/(2p-2)+\varepsilon\right)\delta/(2+\delta)} \sum_{i=1}^{k_n} i^{2(2+\delta-p)(1/p+\varepsilon_1)/(2+\delta)}. \tag{20}$$

Write $\delta_0 = 2(p/(2p-2) - 1 + \varepsilon)^{-1}$. Since $1 < p < 2$, we have $\delta_0 > 0$. Choose $\varepsilon_1 > 0$ sufficiently small, such that

$$0 < \delta < \delta_0 \Rightarrow 2(2+\delta-p)(1/p+\varepsilon_1)/(2+\delta) \le 1 - \eta$$

where $\eta > 0$ does not depend on δ, as long as $0 < \delta < \delta_0$. Because $(p/(2p-2)+\varepsilon)\delta/(2+\delta) < 1$ for $0 < \delta < \delta_0$ and $(p/(2p-2)+\varepsilon)\delta_0/(2+\delta_0) = 1$, one can find $\delta \in (0,\delta_0)$, such that

$$1 - \eta/2 < (p/(2p-2)+\varepsilon)\delta/(2+\delta) < 1.$$

For this δ we have, by (20),

$$\operatorname{var}(S^*_{k_n}) \le Ck_n^{-(1-\eta/2)+1+(1-\eta)+1} \le Ck_n^{-\eta/2}.$$

So we obtain (15) again. Theorem 1 is proved.

Proof of Theorem 2. Part (I): Again we can assume $X_n \ge 0$. Write $Y_n = X_n - EX_n$ and $S^*_n = \sum_{i=1}^n Y_i$. From $\sup E|X_n| < \infty$ we have $\sup_n E|Y_n| < \infty$. Using the same argument employed in proving Theorem 1, we reduce the proof of (11) to that of (15). From (4), (7) and (9),

$$\sum_{n=1}^{\infty} \operatorname{var}(S^*_{k_n})/k_n^2 = \sum_{n=1}^{\infty} k_n^{-2} \sum_{i,j=1}^{k_n} \operatorname{cov}(Y_i,Y_j)$$

$$\le C \sum_{n=1}^{\infty} k_n^{-2} \sum_{i,j=1}^{k_n} \left(\mu(|i-j|)\operatorname{var}(X_i)\operatorname{var}(X_j)\right)^{1/2}$$

$$\le C \sum_{n=1}^{\infty} k_n^{-2} \sum_{i=0}^{k_n} \mu^{1/2}(i) \sum_{i=1}^{k_n} \operatorname{var}(X_i) \tag{21}$$

$$\le C \sum_{n=1}^{\infty} k_n^{-2} \sum_{i=1}^{k_n} \operatorname{var}(X_i)$$

$$\le C \sum_{n=1}^{\infty} \operatorname{var}(X_n)/n^2 < \infty. \tag{22}$$

Part (II) is proved in much the same way as Part (I), only that we replace Ck_n for $\sum_{i=1}^{k_n} \mathrm{var}(X_i)$ and $Ck_n/(\log n)^{1+\varepsilon}$ for $\sum_{i=1}^{k_n} \mu^{1/2}(i)$ in (21) to obtain (22). Part (III) is proved by truncating X_n at n and combining the reasoning above and that of Etemadi [2].

3. AN EXAMPLE

Consider the autoregression model

$$X_n = a_1 X_{n-1} + \ldots + a_m X_{n-m} + e_n, \quad n = 0,\pm 1,\pm 2,\ldots . \tag{23}$$

We want to show that under certain conditions it is true that

$$\lim_{n\to\infty} \sum_{i=1}^{n} X_i/n = 0, \quad \text{a.s.} \tag{24}$$

for any solution of (23). Suppose that the following conditions are satisfied:

1. $\{e_n, n = 0,\pm 1,\ldots\}$ is a sequence of independent real random variables, and

$$Ee_n = 0, \; n = 0,\pm 1,\ldots, \quad \sup_{-\infty<n<\infty} E|e_n|^p = C < \infty \text{ for some } p > 1. \tag{25}$$

where, as before, C is an unimportant constant which is allowed to change.

2. e_n has a density f_n satisfying the Lipschitz condition over R':

$$|f_n(x) - f_n(y)| \le C|x - y|, \quad n = 0,\pm 1,\pm 2,\ldots \tag{26}$$

where C does not depend on n.

3. $a_1, a_2, \ldots, a_m$ are real constants, and the equation $1 - a_1 z - \ldots - a_m z^m = 0$ has all its root outside the unit circle.

Under the condition 1 and 3, the general real solution of (23) has

the form

$$X_n = \sum_{t=0}^{\infty} b_t e_{n-t} + \sum_{j=0}^{J} \rho_j^n \sum_{\ell=0}^{m_j-1} n^{\ell}(\xi_{j\ell} \cos n\omega_j + \eta_{j\ell} \sin n\omega_j) \equiv \tilde{X}_n + X_n^* \qquad (27)$$

where $b_0 = 1$, b_2, b_3, ... are real constants such that

$$|b_t| \leq CH^t, \quad t = 0,1,2,... \quad \text{for some} \quad H \in (0,1). \qquad (28)$$

ρ_j and ω_j, $j = 1,...,J$, are real constants, $0 < \rho_j < 1$, $j = 1,...,J$, $m_1 + ... + m_J = m$, and $\xi_{j\ell}$, $\eta_{j\ell}$, $\ell = 1,...,m_j$, $j = 1,...,J$, are arbitrary random variables. From (25), (27) and (28) it follows that

$$E\tilde{X}_n = 0, \quad n = 0, 1, 2,..., \quad \sup_{-\infty<n<\infty} E|\tilde{X}_n|^p = C < \infty. \qquad (29)$$

Let n, N be positive integers, $n < N$. Define

$$Y_{nN} = \sum_{t=0}^{N-n-1} b_t e_{N-t}, \qquad Z_{nN} = \sum_{t=N-n}^{\infty} b_t e_{N-t}.$$

Since $b_0 = 1$, from (26) it follows that the density g_{nN} of Y_{nN} obeys Lipschitz's condition with the same constant C as in (26). Also

$$\sup\{E|Y_{nN}|^p: 1 \leq n < N < \infty\} = C < \infty. \qquad (30)$$

Now let q_1 be a positive constant, $q_2 = 2q_1$. Define the event

$$D_{nN} = \{|Z_{nN}| \geq (N-n)^{-q_2}\}. \qquad (31)$$

(25) entails $\sup_{-\infty<n<\infty} E|e_n| = C < \infty$. Hence

$$P(D_{nN}) \leq C(N-n)^{q_2} \sum_{t=N-n}^{\infty} H^t \leq C(N-n)^{q_2} H^{N-n}. \qquad (32)$$

Let G be a Borel set in R', h be a constant. G - h is defined as the

set $\{g-h: g \in H\}$. Write $\tilde{G} = G \cap \{u: |u| \le (N-n)^{q_1}\}$, $G^* = G \setminus \tilde{G}$. If $|h| < 1$, we have

$$\begin{aligned}
&|P(Y_{nN} \in G) - P(Y_{nN} \in G - h)| \\
&\quad \le |P(Y_{nN} \in \tilde{G}) - P(Y_{nN} \in \tilde{G} - h)| + P(Y_{nN} \in G^*) + P(Y_{nN} \in G^* - h) \\
&\quad \le \int_{\tilde{G}} |g_{nN}(u) - g_{nN}(u-h)|\,du + P(|Y_{nN}| > (N-n)^{q_1}) + P(|Y_{nN}| > (N-n)^{q_1} - 1) \\
&\quad \le C(N-n)^{q_1} h + C(N-n)^{-q_1} + C[(N-n)^{q_1} - 1]^{-1} \\
&\quad \le C(N-n)^{q_1} h + C(N-n)^{-q_1}. \qquad (33)
\end{aligned}$$

Now let A and B be two Borel sets in R'. We proceed to estimate $|P(\tilde{X}_n \in A, \tilde{X}_N \in B) - P(\tilde{X}_n \in A)P(\tilde{X}_N \in B)|$. From (32), (33) and the independence of $e_1, e_2, \ldots$, we have

$$\begin{aligned}
|P(\tilde{X}_n \in B | e_n, e_{n-1}, \ldots) - P(Y_{nN} \in B)| &= |P(Y_{nN} \in B - Z_{nN} | Z_{nN}) - P(Y_{nN} \in B)| \\
&\le C(N-n)^{-(q_2-q_1)} + C(N-n)^{-q_1} \\
&\le C(N-n)^{-q_1}, \qquad (34)
\end{aligned}$$

when D_{nN} does not occur. But

$$\begin{aligned}
|P(\tilde{X}_N \in B) - P(Y_{nN} \in B)| &= |P(Y_{nN} \in B - Z_{nN}) - P(Y_{nN} \in B)| \\
&= |P(D_{nN}^C)P(Y_{nN} \in B - Z_{nN}) + P(D_{nN})P(Y_{nN} \in B - Z_{nN} | D_{nN}) \\
&\quad - P(Y_{nN} \in B)| \\
&\le P(D_{nN}) + |P(Y_{nN} \in B - Z_{nN} | D_{nN}^C) - P(Y_{nN} \in B)| + P(D_{nN}) \\
&\le 2P(D_{nN}) + C(N-n)^{-q_1} \le C(N-n)^{q_2} H^{N-n} + C(N-n)^{-q_1} \\
&\le C(N-n)^{-q_1}. \qquad (35)
\end{aligned}$$

From (34) and (35) we get

$$|P(\tilde{X}_N \in B | e_n, e_{n-1}, \ldots) - P(\tilde{X}_N \in B)| \leq C(N-n)^{-q_1}$$

when D_{nN} does not occur. If $P(\tilde{X}_n \in B) \geq C(N-n)^{-q_1}$, then from (33) and (35) we obtain

$$P(\tilde{X}_n \in A,\ \tilde{X}_N \in B) \geq [P(\tilde{X}_N \in B) - C(N-n)^{-q_1}][P(\tilde{X}_n \in A) - C(N-n)^{q_2} H^{N-n}]. \quad (36)$$

Also

$$P(\tilde{X}_n \in A,\ X_N \in B) \leq [P(\tilde{X}_N \in B) + C(N-n)^{-q_1}][P(\tilde{X}_n \in A) + C(N-n)^{q_2} H^{N-n}]. \quad (37)$$

From (36) and (37) we have

$$|P(\tilde{X}_n \in A,\ \tilde{X}_N \in B) - P(\tilde{X}_n \in A)P(\tilde{X}_N \in B)| \leq C(N-n)^{-q_1} + C(N-n)^{q_2} H^{N-n}$$

$$+C(N-n)^{q_1} H^{N-n} \leq C(N-n)^{-q_1}, \quad (38)$$

where C does not depend on A, B. (38) is proved when $P(\tilde{X}_n \in B) \geq C(N-n)^{-q_1}$. If $P(\tilde{X}_N \in B) < C(N-n)^{-q_1}$, (38) is trivially true. Therefore we get

$$\alpha(\tilde{X}_n, \tilde{X}_N) \leq C(N-n)^{-q_1}. \quad (39)$$

Now choose $q_1 = p/(2p-2) + 2$. From (39) we see that the condition (2) is satisfied. This, together with (29), gives, by Theorem 1,

$$\lim_{n\to\infty} \sum_{i=1}^{n} \tilde{X}_i/n = 0, \quad \text{a.s.} \quad (40)$$

From the expression of X_n^*, it is readily seen that

$$\lim_{n\to\infty} \sum_{i=1}^{n} X_i^*/n = 0, \quad \text{a.s.} \quad (41)$$

From (27), (40) and (41), we obtain (24).

The conclusion (40) does not follow from the ergodic theorem of stationary process, since $\{e_n\}$ is not assumed to be identically distributed, so $\{X_n\}$ may not be a strictly stationary process.

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